Precalculus UNIT 2 REVIEW Key (your name here!) loves complex numbers

Write an equation for the line between the two points: in 3 forms: point-slope, slope-intercept, general

1. $f(-3)=-11$ *and* $f(7)=3$

$m = \frac{3-(-11)}{7-(-3)} = \frac{14}{10} = \frac{7}{5}$

$y-3=\frac{7}{5}(x-7)$ or $y+11=\frac{7}{5}(x+3)$

$y=\frac{7}{5}x-\frac{49}{5}+\frac{15}{5}$

$y=\frac{7}{5}x-\frac{43}{5}$

$\frac{7}{5}x-y-\frac{43}{5}=0$

$7x-5y-43=0$

Find the vertex and line of symmetry and y-intercept of these quadratics(w/o calc--show work):

2. $y=-3(x+5)^2-9$

vertex $(-5,-9)$

line of sym $x=-5$

y-int $y=-3(0+5)^2-9$

$=-3(25)-9=-75-9$

$(0,-84)$

3. $y=-x^2+7x+18$

line of sym $=\frac{-b}{2a}=\frac{-7}{2(-1)}=\frac{7}{2}$

vertex $y=-(\frac{7}{2})^2+7(\frac{7}{2})+\frac{18}{1}=-\frac{49}{4}+\frac{49}{2}+\frac{18}{1}$

$=\frac{49}{4}+\frac{18}{1}\cdot\frac{4}{4}=\frac{121}{4}$ $(\frac{7}{2},\frac{121}{4})$

y-int $y=0+0+18$ $(0,18)$

Write the equation in vertex form by completing the square:

4. $y=x^2-14x+1$

$y=x^2-14x+(\frac{-14}{2})^2+1-49$

$y=x^2-14x+49-48$

$y=(x-7)^2-48$

5. $y=2x^2+5x+16$

$y=2(x^2-\frac{5}{2}x+8)$

$y=2(x^2-\frac{5}{2}x+(\frac{-5/2}{2})^2+8-\frac{25}{16})$

$y=2(x-\frac{5}{4})^2+\frac{206}{16}$

$\frac{5}{2}\div\frac{2}{1}=\frac{5}{2}\cdot\frac{1}{2}(\frac{5}{4})$

$128\frac{8}{16}-\frac{25}{16}$

$2(\frac{103}{16})$

$\frac{16}{128}$

Write the equation in standard form by "FOIL"ing:

6. $y=-\frac{3}{2}(x-6)^2+1$

$y=-\frac{3}{2}(x^2-12x+36)+1$

$y=-\frac{3}{2}x^2+\frac{36}{2}x-\frac{108}{2}+1$

$y=-\frac{3}{2}x^2+18x-53$

Given the vertex and a point, write an equation for the quadratic equation:

6.5. vertex $(3,-11)$ $h\ k$ and point $(5,4)$ $x\ y$

$y=a(x-h)^2+k$

$4=a(5-3)^2-11$

$4=a(2)^2-11$

$\frac{15}{4}=\frac{4a}{4}$

$a=\frac{15}{4}$

$y=\frac{15}{4}(x-3)^2-11$

State if function is a polynomial, if yes, what degree and leading coefficient, if no, why not?

7. $y=-2\sqrt{x}-\frac{3}{4}+x^5$

No, exponent is fractional.

8. $y=3x^4-\frac{2}{7}x$ yes.

9. You dive off the 3 foot diving board at City Park, with an initial vertical velocity of 5 [redacted] per second. $h=\frac{-1}{2}gt^2+v_0t+h_0$ $g=32ft/s^2$

a. Write an equation to represent the situation. $h(t)=-16t^2+5t+3$

b. Find maximum height you reach *using algebra*.

vertex → y value

line of sym $x=\frac{-5}{2(-16)}=\frac{-5}{-32}=.15625$

$h(.15625)=-16(.15625)^2+5(.15625)+3$

$h(.15625)=3.390625$

max height ≈ 3.4 feet

c. Find the time you hit water *using algebra*.

Quad Form ← $h(t)=0$ $0=-16t^2+5t+3$

$x=\frac{-5\pm\sqrt{25-4(-16)(3)}}{2(-16)}\rightarrow\frac{-5\pm\sqrt{217}}{-32}$ $<\begin{matrix}-.304\text{ s}\\ .6166\text{ sec}\end{matrix}$

Hit H_2O at $t=.625$ sec

d. Given the velocity equation $v(t)=-gt+v_0$. Find the velocity when you hit water.

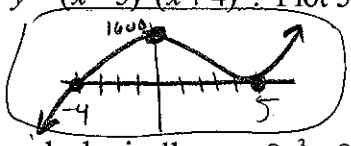
$v(.62)=-32(.62)+5$

$-19.84+5=-14.84$ ft/s

…nce varies directly with time. I go 45 miles in 3 hours. Traveling at the same rate, how long will it take me to go …s? $D=kt$ $45=k(3)$ $k=15$ $D=15t$ $10=15t$ $t=\frac{10}{15}=\frac{2}{3}$ hours = 40 minutes

11. Make a sketch of the graph $y=(x-5)^2(x+4)^3$. Plot 3 exact points.

Degree 5



$y=(0-5)^2(0+4)^3$
$y=(25)(64)$
$y=1600$ ← y-int

12. Find the zeros of the function algebraically. $y=8x^3-8x^2-16x$

Try Factor-First
$y=8x(x^2-x-2)$
$y=8x(x-2)(x+1)$
zeros $x=0,2,-1$

13. Use long division to divide $\frac{x^4-6x^3+-3x+5}{x-2}$.

$x^3-4x^2-8x-19-\frac{33}{x-2}$

$x-2\overline{)\,1x^4-6x^3+0x^2-3x+5}$
$-(x^4-2x^3)$
$-4x^3+0x^2$
$-(-4x^3+8x^2)$
$-8x^2-3x$
$-(-8x^2+16x)$
$-19x+5$
$-(-19x+38)$
-33

14. Write out the list of all possible rational zeros for $f(x)=3x^4+3x^3-18$.

factors last coef / factors first coef $=\frac{\pm1,\pm2,\pm3,\pm6,\pm9,\pm18}{\pm1,\pm3}$

$=\pm1,\pm2,\pm3,\pm6,\pm9,\pm18,\pm\frac{1}{3},\pm\frac{2}{3}$

6.

15. Find all the real zeros of the function using synthetic division and/or factoring or quadratic equation for

$f(x)=(x^2-9)(x^2-4)$ easiest
$f(x)=(x+3)(x-3)(x+2)(x-2)$
$x=3,-3,2,-2$

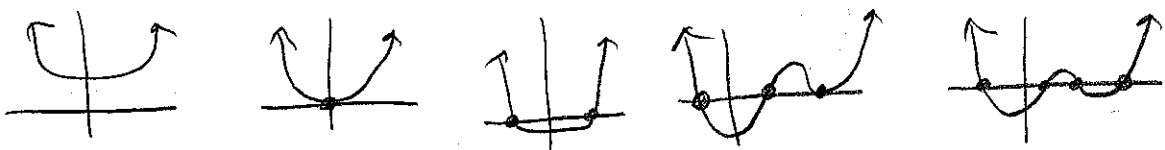
2 | 1 0 −13 0 36 Deg 4
↓ 2 4 −18 −36
1 2 −9 −18 0 Rem Deg 3

−2 | 1 2 −9 −18 0 Rem Deg 3
↓ −2 0 18
1 0 −9 0 Rem Deg 2

3 | 1 0 −9 0 Rem Deg 2
↓ 3 9
1 3 0 Rem Deg 1

+3 | 1 3 Deg 1
↓ 3
1 0 Rem ← x^0 Deg 0

16. Draw a sketch of a quartic function that has no zeros one zero, two zeros and three zeros and 4 zeros.



17. Find the y value when $x=-2$ for $f(x)=-5x^3+9x^2-x+1$ using synthetic division.

−2 | −5 9 −1 1
↓ 10 −38 78
−5 19 −39 79

Check
$f(-2)=-5(-2)^3+9(-2)^2-(-2)+1$

18. Simplify $\frac{(-2i)(1-6i)}{(1+6i)(1-6i)}$

$=\frac{-2i+12i^2}{1-6i+6i-36i^2}=\frac{-2i+12(-1)}{1-36(-1)}=\frac{-12-2i}{37}=\frac{-12}{37}-\frac{2i}{37}$

19. Simplify $(5-\sqrt{-16})(-2+4i^5)$

$-10+20i^5+2\sqrt{-16}-4\sqrt{-16}\,i^5$
$-10+20(-1)(-1)i+2\sqrt{-1}\sqrt{16}-4\sqrt{-1}\sqrt{16}(-1)(-1)i$

$-10+20i+8i-16i^2$
$=-10+28i+16$
$=6+28i$

e $x^2+x+9=4x-2$ and find your complex solutions.

$x^2-3x+11=0$

$x=\frac{3\pm\sqrt{9-4(1)(11)}}{2(1)}$

$x=\frac{3\pm\sqrt{-35}}{2}$

$x=\frac{3}{2}\pm\frac{\sqrt{35}i}{2}$

21. Write the polynomial in standard form, identify the zeros and x-intercepts: $f(x)=(x-5)(x-\sqrt{2}i)(x+\sqrt{2}i)$

$(x-\sqrt{2}i)(x+\sqrt{2}i)$ (x^2-2i^2)

(x^2+2)

$(x-5)$

$-5x^2 \quad -10$

$x^3 \quad 2x$

$f(x)=x^3-5x^2+2x-10$

Zeros: $x=5, \sqrt{2}i, -\sqrt{2}i$

x-int: $x=5$

22. Write the standard form of a polynomial function of degree 5 whose zeros include x = 1, x = 1+2i, x = 1+ i .

Expand. $f(x)=(x-1)(x-(1+2i))(x-(1-2i))(x-(1+i))(x-(1-i))$

conj.

$f(x)=(x-1)[x^2-x(1-2i)-(1+2i)x+(1+2i)(1-2i)][x^2-x(1-i)-(1+i)x+(1+i)(1-i)]$

$f(x)=(x-1)(x^2-x+2ix-x-2ix+1-4i^2)(x^2-x+xi-x-ix+1-i^2)$

$=(x-1)(x^2-2x+5)(x^2-2x+2)$

work $=(x^3-3x^2+7x-5)(x^2-2x+2)$

$=x^5-5x^4+15x^3-25x^2+24x-10$

23. Factor $f(x)=x^5-3x^4-5x^3+5x^2-6x+8$ and write f(x) in its linear factorization.

Look calc-try $x=-2$, $x=1$, $x=4$

Deg 5

-2 | 1 −3 −5 +5 −6 8

↓ −2 10 −10 10 −8

1 −5 5 −5 4 0 Rem Deg 4

1 | 1 −4 1 −4

↓ 1 −4 1 −4

1 −4 1 −4 0 Rem Deg 3

4 | 1 0 4

↓ 4 0 4

1 0 1 0 Rem Deg 2

$x^2+1=0$

$x^2=-1$

$x=\pm\sqrt{-1}$

$x=\pm i$

$f(x)=(x+2)(x-1)(x-4)(x-i)(x+i)$

24. The complex number z = 1- 2i is a zero of $f(x)=4x^4+17x^2+14x+65$. Use syn. Div. to find the remaining zeros and write it in its linear factorization.

Deg 4

$1-2i$ | 4 0 17 14 65

↓ $4-8i$ $-12-16i$ $-27-26i$ -65

4 $4-8i$ $5-16i$ $-13-26i$ 0 Rem Deg 3

$1+2i$ conjugate | 4 $4-8i$ $5-16i$ $-13-26i$

↓ $4+8i$ $8+16i$ $13+26i$

4 8 13 0 Rem Deg 2

$4x^2+8x+13=0$

Quad Form

$x=\frac{-8\pm\sqrt{64-4(4)(13)}}{2(4)}$

$=\frac{-8\pm\sqrt{-144}}{8}=\frac{-8\pm12i}{8}=-1\pm\frac{3i}{2}$

$f(x)=(x-(1-2i))(x-(1+2i))(x-(-1+\frac{3}{2}i))(x-(-1-\frac{3}{2}i))$

25. Write $f(x)=3x^5-2x^4+6x^3-4x^2-24x+16$ as a product of linear and quadratic factors, ~~each with real~~ coefficients.

$\pm1,\pm2,\pm4,\pm8,\pm16$ / $\pm1,\pm3$

Look calc-try $2/3$

$2/3$ | 3 −2 6 −4 −24 16 Deg 5

↓ 2 0 4 0 −16

3 0 6 0 −24 0 Rem Deg 4

no more rational #.

$0=3x^4+6x^2-24$ FACTOR!!

$\div 3$

Study ~~2.7 and 2.8~~

$=3(x-\frac{2}{3})(x-\sqrt{2})(x+\sqrt{2})(x^2+4)$

$=(3x-2)(x-\sqrt{2})(x+\sqrt{2})(x^2+4)$

x^2-2x+5

(×) $x-1$

$-x^2+2x-5$

x^3-2x^2+5x

(x^3-3x^2+7x-5)

(+) x^2-2x+2

$x^5-3x^4+7x^3-5x^2$

$-2x^4+6x^3-14x^2+10x$

$2x^3-6x^2+14x-10$

(+)

$x^5-5x^4+15x^3-25x^2+24x-10$

$3x^4+6x^2-24$ FACTOR

$3(x^4+2x^2-8)$

$3(x^2-2)(x^2+4)$

$3(x+\sqrt{2})(x-\sqrt{2})(x^2+4)$

could break into 2 complex zeros:

$(x-2i)(x+2i)$

But Q didn't ask for that.