Calculus—REVIEW for quiz
Unit 5.1-5.3

Name Key - Posted

Differentiate

1. $y=\ln(x^3-6x)$

$$\boxed{y'=\frac{1}{x^3-6x}(3x^2-6)}$$

2. $y=(\ln x)^3$

$$\boxed{y'=3(\ln x)^2\cdot\frac{1}{x}}$$

3. ~~$y=x3^{\cos(4x)}$~~

4. $y=\ln\frac{x(x^2+1)^2}{\sqrt{2x^3-1}}=\ln x+2\ln(x^2+1)-\frac{1}{2}\ln(2x^3-1)$

$$y'=\frac{1}{x}+\frac{2}{x^2+1}\cdot 2x-\frac{1}{2}\cdot\frac{1}{2x^3-1}\cdot 6x^2$$

$$\boxed{y'=\frac{1}{x}+\frac{4x}{x^2+1}-\frac{3x^2}{2x^3-1}}$$

5. $y=\sqrt{\frac{x^2-1}{x^2+1}}$ use logarithmic differentiation

$\ln y = \ln\sqrt{\frac{x^2-1}{x^2+1}}$

$$\ln y=\frac{1}{2}\left(\ln(x+1)+\ln(x-1)-\ln(x^2+1)\right)$$

$$\cancel{y}\,\cancel{\frac{1}{y}}\boxed{\frac{dy}{dx}=\frac{1}{2}\left(\frac{1}{x+1}+\frac{1}{x-1}-\frac{1}{x^2+1}\cdot 2x\right)\cdot\left(\sqrt{\frac{x^2-1}{x^2+1}}\right)}$$

6. $\int_0^{x^2}\sec(t^3-6t)dt$

FTC2 $\boxed{\sec(x^6-6x^2)\cdot 2x}$

6.75 $y=x^x$ use logarithmic differentiation

$\ln y = \ln x^x$

$\ln y = x\ln x$

$$\cancel{y}\,\cancel{\frac{1}{y}}\boxed{\frac{dy}{dx}=\left(x\left(\frac{1}{x}\right)+\ln x\right)x^x}$$

7. Use implicit differentiation to find the equation of the tangent line to the graph at the given point:

$y^2+\ln(xy)=2$ at $(e,1)$.

$$2y\frac{dy}{dx}+\frac{1}{xy}\left(x\frac{dy}{dx}+y\right)=0$$

$$2y\frac{dy}{dx}+\frac{1}{y}\frac{dy}{dx}+\frac{1}{x}=0$$

$$\frac{dy}{dx}=\frac{-\frac{1}{x}}{2y+\frac{1}{y}}$$

$$m=\frac{-\frac{1}{e}}{2+1}=-\frac{1}{3e}$$

$$\boxed{y-1=-\frac{1}{3e}(x-e)}$$

Evaluate the integrals

8. $\int\frac{5}{x}dx$

$$\boxed{5\ln|x|+C}$$

9. Division $\int\frac{x^2-3x+2}{x+1}dx$

$$\begin{array}{r} x-4 \\ x+1\;\overline{)\;x^2-3x+2} \\ -(x^2+x) \\ -4x+2 \\ -(-4x-4) \\ 6 \text{ Rem} \end{array}$$

$$\int x-4+\frac{6}{x+1}=\boxed{\frac{1}{2}x^2-4x+6\ln|x+1|+C}$$

10. $\int\tan 2x\,dx$

$u=2x$

$\frac{du}{2}=\cancel{2}dx/\cancel{2}$

$$\boxed{-\frac{1}{2}\ln|\cos 2x|+C}$$

11 ~~$\int(\cos x)4^{(\sin x)}dx$~~

12. $\int_{\pi/2}^{2\pi/3}(\cot t-\cos t)dt$

$$\ln|\sin t|-\sin t\Big]_{\pi/2}^{2\pi/3}$$

$$\ln|\sin\tfrac{2\pi}{3}|-\sin\tfrac{2\pi}{3}-\left(\ln|\sin\tfrac{\pi}{2}|-\sin\tfrac{\pi}{2}\right)$$

$$\boxed{\ln\tfrac{\sqrt{3}}{2}-\tfrac{\sqrt{3}}{2}-0+1}$$

13. $\int_0^{\pi/3}\tan^2(3t)dt$ use a trig id!!!

$u=3t$

$\frac{du}{3}=\frac{\cancel{3}dt}{\cancel{3}}$

$$\int_0^{\pi/3}\sec^2(3t)-1\,dt$$

$$=\frac{1}{3}\left(\tan 3t-t\right]_0^{\pi/3}$$

$$=\frac{1}{3}\left(0-\frac{\pi}{3}-(0-0)\right)$$

$$=\boxed{\pi/9}$$

Find $\left(f^{-1}\right)'(a)$ for the function f and the given number a.

14. $f(x)=\frac{1}{27}(x^5+2x^3), a=-11$

$$(f^{-1})'(-11)=\frac{1}{f'(-3)}=\frac{1}{\frac{1}{27}(5(-3)^4+6(-3)^2)}=\boxed{\frac{1}{17}}$$

* a of inverse is b of original

$-11=\frac{1}{27}(x^5+2x^3)$ $x=-3$

* use calculator

15. $f(x)=\sqrt{x-4}, a=2$

Find b

$2=\sqrt{x-4}$

$4=x-4$

$8=x$

$$(f^{-1})'(2)=\frac{1}{f'(8)}=\frac{1}{\frac{1}{2}(x-4)^{-1/2}}\Bigg|_{x=8}=\frac{2}{4^{-1/2}}=\boxed{4}$$

16. Suppose that the line $y=7-4(x-3)$ is tangent to the graph of a function f at the point (3, 7). Let f^{-1} be the inverse of f. Write an equation of the line tangent to the graph of f^{-1} at the point (7, 3).

original $f(3)=7, f'(3)=-4$

Inverse $f^{-1}(7)=3, f^{-1}{}'(7)=-\frac{1}{4}$

$$\boxed{y-3=-\frac{1}{4}(x-7)}$$

$$\boxed{(f^{-1})'(a)=\frac{1}{f'(b)}}$$

old way: find inverse of line

$y=7-4(x-3)$

$x=7-4(y-3)$

$\frac{x-7}{-4}=\frac{\cancel{-4}(y-3)}{\cancel{-4}}$

$$\boxed{-\frac{1}{4}(x-7)+3=y}$$

same.