Student key

Calculus—Review Unit 5 Name Key

Differentiate

1. $y = 5^{\tan x}$

$y' = (\ln 5)5^{\tan x} \cdot \sec^2 x$

2. $y = \ln(x^4 \sin x)$ used log props first.

$y = \ln x^4 + \ln \sin x$

$y' = \frac{4}{x} + \frac{1}{\sin x} \cdot \cos x = \frac{4}{x} + \cot x$

3. $y = \arcsin(x^2 - 3)$

$y' = \frac{1 \cdot 2x}{\sqrt{1-(x^2-3)^2}}$

4. $y = \operatorname{arcsec}(e^x)$

$y' \frac{1}{|e^x|\sqrt{(e^x)^2-1}} \cdot e^x$

5. $y = \frac{x^2\sqrt[3]{x-4}}{3x-1}$ use logarithmic differentiation.

$\ln y = 2\ln x + \ln \sqrt[3]{x-4} - \ln(3x-1)$

$\frac{dy}{dx} = \left[2 \cdot \frac{1}{x} + \frac{1}{3} \cdot \frac{1}{(x-4)} - \frac{1}{3x-1} \cdot 3\right] \cdot \left(\frac{x^2\sqrt[3]{x-4}}{3x-1}\right)$

6. $\frac{d}{dx}\int_1^{\ln x} (e^t - t^2)dt$ $\left(e^{\ln x} - (\ln x)^2\right)\frac{1}{x}$

7. Find the equation of the tangent line to the graph at the given point: $y = \arcsin x$ for $\left(\frac{1}{2}, \frac{\pi}{6}\right)$

$y' = \frac{1}{\sqrt{1-x^2}}$ $y'(\frac{1}{2}) = \frac{1}{\sqrt{1-(1/2)^2}} = \frac{1}{\sqrt{3/4}}$

$y - \frac{\pi}{6} = \frac{1}{\sqrt{3/4}}(x - \frac{1}{2})$

Evaluate the integrals

8. $\int \frac{1}{2x+7}dx$ $u = 2x+7$, $\frac{du}{2} = \frac{2dx}{2}$

$\frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln|2x+7| + C$

9. $\int \tan 5x dx$ $-\frac{1}{5}\ln|\cos(5x)| + C$

10. $\int x6^{x^2} dx$ $u = x^2$, $\frac{du}{2} = \frac{2x dx}{2}$

$\frac{1}{2}\int 6^u du$

$\frac{1}{2} \cdot \frac{1}{\ln 6} 6^{x^2} + C$

11. $\int (\sec x \tan x)e^{\sec x}dx$ $u = \sec x$, $du = \sec x \tan x\, dx$

$\int e^u du = e^u + C = e^{\sec x} + C$

12. $\int \frac{1}{x\ln x}dx$ $u = \ln x$, $du = \frac{1}{x}dx$

$\int \frac{1}{u}du = \ln|u| + C = \ln|\ln x| + C$

13. $\int (1+\cot^2 x)dx$ use trig id!! $\sin^2 x + \cos^2 x = 1$ $(\div \sin^2 x)$ $1 + \cot^2 x = \csc^2 x$

$\int \csc^2 x\, dx$

memorize $= -\cot x + C$

14. $\int \frac{1}{x^2+16}dx$ $a = 4$

$\frac{1}{4}\arctan \frac{x}{4} + C$

15. $\int \frac{e^x}{\sqrt{25-(e^x)^2}}dx$ $u = e^x$, $du = e^x dx$ $a = 5$

$\int \frac{1}{\sqrt{1-u^2}}du = \arcsin\left(\frac{e^x}{5}\right) + C$

Find $(f^{-1})'(a)$ for the function f and the given number a. Evaluate without a calculator

$(f^{-1})'(a) = \frac{1}{f'(b)}$

16. $f(x) = x^3 + 2x - 1, a = 2$

(1) $2 = x^3 + 2x - 1$

$0 = x^3 + 2x - 3$ (calculator)

$x = 1$ which is our "b"

(2) $f'(x) = 3x^2 + 2$

$f'(1) = f'(b) = 3(1)^2 + 2 = 5$

$\frac{1}{5}$

17. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\sqrt{3}$, 60, 2

$-\pi/3$

think restricted domain

18. $\cos^{-1}\left(-\frac{1}{2}\right)$

2, 120, -1

$\frac{2\pi}{3}$

19: Evaluate $\int_{-\frac{1}{2}}^{0} \frac{x-3}{\sqrt{9-x^2}}dx$ Skip

separate $\frac{3+x}{x}$ $\frac{3}{x} + \frac{x}{x}$

$\int \frac{x}{\sqrt{9-x^2}} - 3\int \frac{1}{\sqrt{9-x^2}}$ $u = x$, $du = dx$, $a = 3$

$u = 9 - x^2$

$\frac{du}{-2} = \frac{-2x dx}{-2}$

$-\frac{1}{2}\int \frac{1}{\sqrt{u}}du$

$- 3\left(\arcsin\left(\frac{x}{3}\right)\right)\Big]$

Finish ... we'll learn later

Did in class.