Calculus BC
Unit 8 REVIEW—SHOW WORK!!!

Good Luck To_____

Evaluate using the method stated:

1. $\int \frac{x-1}{\sqrt{36-x^2}}dx$

2. $\int y^2 \sin 3y dy$

3. $\int \frac{5}{(9x^2+4)}dx$

4. $\int \frac{xdx}{2x^2-5x-12}$.

5. show work to determine converge or diverge $\int_1^\infty \frac{5}{x^2+1}dx$

Evaluate:

6. $\int \frac{x^2+3}{x}dx$

7. $\int \sin^3 x \cos^2 x dx$

8. $\int \ln x dx$

9. Evaluate $\lim_{x\to 0} \frac{1-\cos x}{3x^2}$

10. Evaluate $\lim_{x\to 0} \frac{e^{x^2}-1}{2x^2}$

11. Evaluate $\int e^x \cos 2x dx$

12. Evaluate $\lim_{x\to 0^+} x^x$

(12) $y = \lim_{x\to 0^+} x^x$

$\ln y = \lim_{x\to 0^+} x \cdot \ln x = 0 \cdot -\infty$

$\ln y = \lim_{x\to 0^+} \frac{\ln x}{1/x} = \frac{-\infty}{\infty}$

$L'Hôp$

$\ln y = \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+} \frac{-x^2}{x}$

$= \lim_{x\to 0^+} -x = 0$

$\ln y = 0$

$y = 1$

Key

1) $\int \frac{x-1}{\sqrt{36-x^2}}\,dx$

$\int \frac{x}{\sqrt{36-x^2}} - \int \frac{1}{\sqrt{36-x^2}}\,dx$

$u = 36-x^2$ $\quad a=6$ $\quad u = x$

$du = -2x\,dx$ $\quad du = dx$

$-\frac{1}{2}\int u^{-1/2}\,dx - \int \frac{1}{\sqrt{a^2-u^2}}\,du$

$-\frac{1}{2}\left[\frac{2}{1}u^{1/2} - \sin^{-1}\left(\frac{u}{a}\right)\right] + C$

$\boxed{-\sqrt{36-x^2} - \sin^{-1}\left(\frac{x}{6}\right) + C}$

2) $\int y^2 \sin 3y\,dy$

$+\ y^2$	$\sin 3y$
$-\ 2y$	$-\frac{1}{3}\cos 3y$
$+\ 2$	$-\frac{1}{9}\sin 3y$
$-\ 0$	$\frac{1}{27}\cos 3y$

$\boxed{-\frac{y^2}{3}\cos 3y + \frac{2y}{9}\sin 3y + \frac{2}{27}\cos 3y + C}$

3) $\int \frac{5}{9x^2+4}\,dx$ $\quad a^2 = 4$ $\quad a = 2$ $\quad u^2 = 9x^2$ $\quad u = 3x$

$\frac{du}{3} = \frac{3\cdot dx}{3}$

$\frac{5}{3}\int \frac{du}{u^2+a^2}$

$\frac{5}{3}\left[\frac{1}{2}\tan^{-1}\left(\frac{u}{a}\right)\right] + C$

$\boxed{\frac{5}{6}\arctan\left(\frac{3x}{2}\right) + C}$

4) $\int \frac{x}{2x^2-5x-12}\,dx = \int \frac{A}{2x+3} + \frac{B}{x-4} \rightarrow \int \left(\frac{3/11}{2x+3} + \frac{4/11}{x-4}\right)dx$

$u = 2x+3$

$\frac{du}{2} = \frac{2\,dx}{2}$

$\frac{1}{2}\cdot\frac{3}{11}\ln|2x+3| + \frac{4}{11}\ln|x-4| + C$

24, -5, -8, 3

$2x^2 - 8x + 3x - 12$

$2x(x-4) + 3(x-4)$

P.F.D $\quad x = A(x-4) + B(2x+3)$

Let $x = 4$ $\quad 4 = 0 + B(11)$ $\quad B = \frac{4}{11}$

Let $x = -\frac{3}{2}$ $\quad -\frac{3}{2} = A\left(-\frac{11}{2}\right) + 0$ $\quad \frac{3}{11} = A$

5. $\int_1^\infty \frac{5}{x^2+1}\,dx$

$\lim_{b\to\infty} \int_1^b \frac{5}{x^2+1}\,dx = \lim_{b\to\infty} [5\cdot\arctan x]_1^b$

$\lim_{b\to\infty} [5\tan^{-1}(b) - 5\tan^{-1}(1)]$

$\sim \frac{5\pi}{2\cdot 2} - 5\frac{\pi}{4} = \frac{5}{4}\pi$

6. $\int \frac{x^2+3}{x}\,dx$

$= \int \left(x + \frac{3}{x}\right) dx = \frac{1}{2}x^2 + 3\ln|x| + C$

7. $\int \sin^3 x \cos^2 x\,dx \Rightarrow \int \sin^2 x \cos^2 x \sin x\,dx$

$= \int (1-\cos^2 x)\cos^2 x \cdot \sin x\,dx = \int (\cos^2 x - \cos^4 x)\sin x\,dx$

$u = \cos t$

$du = -\sin x\,dx$

$\int (u^2 - u^4)\,du = \frac{1}{3}u^3 - \frac{1}{5}u^5 + C = \frac{1}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$

8. $\int \ln x\,dx \Rightarrow \begin{matrix} u = \ln x & dv = dx \\ du = \frac{1}{x}dx & v = x \end{matrix}$

$x\ln x - \int x\cdot\frac{1}{x}\,dx = x\ln x - \int dx$

$= x\ln x - x + C$

9. $\lim_{x\to 0} \frac{1-\cos x}{3x^2} \overset{0/0}{=} \lim_{x\to 0} \frac{+\sin x}{6x} \overset{0/0}{=} \lim_{x\to 0} \frac{\cos x}{6} = \frac{1}{6}$

10. $\lim_{x\to 0} \frac{e^{x^2}-1}{2x^2} \overset{0/0}{=} \lim_{x\to 0} \frac{2xe^{x^2}}{4x} \overset{0/0}{=} \lim_{x\to 0} \frac{2x(2xe^{x^2}) + e^{x^2}\cdot 2}{4}$

$= \frac{2}{4} = \frac{1}{2}$

11. $\int e^x \cos 2x\,dx$

$u = e^x \quad dv = \cos 2x\,dx$

$du = e^x dx \quad v = \frac{1}{2}\sin 2x$

$\frac{1}{2}e^x \sin 2x - \int \frac{1}{2}e^x \sin 2x\,dx$

$u = e^x \quad dv = \sin 2x\,dx$

$du = e^x \quad v = -\frac{1}{2}\cos 2x$

$\frac{1}{2}e^x \sin 2x - \frac{1}{2}\left[-\frac{1}{2}e^x\cos 2x - \int -\frac{1}{2}e^x \cos 2x\,dx\right]$

$\int e^x \cos 2x\,dx = \frac{1}{2}e^x \sin 2x + \frac{1}{4}e^x \cos 2x + \frac{1}{4}\int e^x \cos 2x\,dx$

$\frac{3}{4}\int e^x \cos 2x\,dx = \frac{1}{2}e^x \sin 2x + \frac{1}{4}e^x \cos 2x$

$\int e^x \cos 2x\,dx = \frac{2}{3}e^x \sin 2x + \frac{1}{3}e^x \cos 2x + C$