Calculus
Review Unit 9 2012

Good Luck to Key

1. Determine convergence, if converges, find the sum: a) $\sum_{n=0}^{\infty} 3\left(\frac{2}{5}\right)^n$ b) $\sum_{n=0}^{\infty} 4\left(\frac{\pi}{3}\right)^n$ c) $\sum_{n=0}^{\infty} \frac{3^{n+1}}{5^n}$ $3 \cdot \frac{3^n}{5^n}$

a) $\frac{3}{1-2/5} = \frac{3/1}{3/5} = 5$ c) $\frac{3}{1-3/5} = \frac{3}{2/5} = \frac{15}{2}$ a) conv $|r| = \left|\frac{2}{5}\right| < 1$ b) DIV $|r| = \frac{\pi}{3} \nless 1$ c) conv $r = \left|\frac{3}{5}\right| < 1$

2. Investigate a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$ and b) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{5+n^2}$ for convergence or divergence. If it converges, determine whether it converges conditionally, or absolutely.

a) $\lim_{n\to\infty} \frac{1}{\sqrt{n}} \to 0$. conv. $\sum \frac{1}{\sqrt{n}}$ Div by p-series

$\frac{1}{\sqrt{n+1}} \le \frac{1}{\sqrt{n}}$ ✓ $\therefore$ Converge conditionally

b) $\lim_{n\to\infty} \frac{1}{5+n^2} = 0$ $\frac{1}{5+(n+1)^2} \le \frac{1}{5+n^2}$ ✓ $\sum \frac{1}{5+n^2}$ conv by comp. to p-series. absolute conv

3. Determine the convergence or divergence of a) $\sum_{n=0}^{\infty} \frac{2n}{n^2+3}$ and b) $\sum_{n=0}^{\infty} \frac{2n^2}{n^2+7}$. Show your work.

a) compare to $\sum \frac{n}{n^2} = \sum \frac{1}{n}$ Div.

$\lim_{n\to\infty} \frac{2n^2}{n^2+3} = 2$ $\therefore$ positive finite $\therefore$ $\boxed{\text{Diverge}}$

b) $\lim_{n\to\infty} a_n = 2 \neq 0$. $\boxed{\text{Diverge by } n^{th} \text{ term test}}$

4. Which of the following series converge?

i. $\sum_{n=0}^{\infty} \left(\frac{5}{2^n}\right)$ geom. converge $|r| = \left|\frac{1}{2}\right| < 1$

ii. $\sum_{n=0}^{\infty} \left(\frac{3^n n!}{(2n)! 5^n}\right)$ Ratio

$\lim_{n\to\infty} \frac{3^n 3^1 (n+1) n!}{(2(n+1))! 5^n 5^1} \cdot \frac{(2n)! 5^n}{3^n \cdot n!}$ $(2n+2)(2n+1)(2n)!$ $= \lim_{n\to\infty} \frac{3(n+1)}{(2n+2)(2n+1)5} = 0$ conv. Always

iii. $\sum_{n=1}^{\infty} \frac{\ln n}{n} > \sum \frac{1}{n}$ Direct compare to $\sum \frac{1}{n}$ Diverge

5. Determine the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-4)^n}{n3^n}$. Endpoints!!

Ratio $\lim_{n\to\infty} \left| \frac{(x-4)^n (x-4)}{(n+1) 3^n 3^1} \cdot \frac{n 3^n}{(x-4)^n} \right| = \lim_{n\to\infty} \left| \frac{x-4}{3} \cdot \frac{n}{(n+1)} \right| = \left| \frac{x-4}{3} \right| < 1$ $\boxed{1 \le x < 7}$

$|x-4| < 3$ TESTS $\longrightarrow$

6. Write the Maclaurin polynomial to degree 3, and write the general series form for $f(x) = xe^{3x}$.

$e^x = \sum \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \dots$

$e^{3x} = \sum \frac{(3x)^n}{n!} = 1 + 3x + \frac{(3x)^2}{2!} + \dots$

$xe^{3x} \approx x + 3x^2 + \frac{3^2 x^3}{2!} + \frac{3^3 x^4}{3!}$

$= \sum_{n=0}^{\infty} \frac{3^n x^{n+1}}{n!}$

7. Show your work to generate (or construct) the Taylor series for $f(x) = \ln(x+1)$ centered at c=0.

$f(x) = \ln(x+1)$

$f'(x) = \frac{1}{(x+1)} = (x+1)^{-1}$

$f''(x) = \frac{-1}{(x+1)^2} = -(x+1)^{-2}$

$f'''(x) = -1 \cdot -2 (x+1)^{-3} = \frac{2}{(x+1)^3}$

$f^{(4)}(x) = 1 \cdot 2 \cdot -3 (x+1)^{-4}$

$f^{(n)}(x) = (-1)^{n+1} (n-1)! (x+1)^{-n}$

$\ln(x+1) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} \cancel{(n-1)!}}{n \cancel{(n-1)!}} (x-0)^n$

$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$

$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$

8. Find the third Taylor Polynomial for $f(x) = e^{-x}$ at a=0. Use this to approximate $f(1)$ and determine the upper bound for the error of the approximation.

$e^{-x} = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} \approx 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}$ $e^{-1} \approx 1 - 1 + \frac{1}{2} - \frac{1}{6}$

9. Try AP multiple choice and free response again.

$f = e^{-x}$	1
$f' = -e^{-x}$	-1
$f'' = e^{-x}$	1
$f''' = -e^{-x}$	-1
$f^{(4)} = e^{-x}$	1

check $e^{-1} = .36787$ error $= .0345$

$f(1) = e^{-1} \approx .3333 = \frac{1}{3}$

$|f(1) - P_3(1)| \le$ alt series next term $\left|\frac{1}{4!}\right| = .04166$

$= \frac{1}{e^x}$ $\max_{0 \le x \le 1} \frac{1}{e^x}$ $\max = \frac{1}{e^0} = 1$ $\frac{1}{e^0} > x > \frac{1}{e^1}$

Taylor Lagrange $\frac{1}{4!}(1)^4$ Same!